Tax competition, location, and horizontal foreign direct investment

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IFIR Working Paper No. 2006-08

February 2006

*We thank Oscar Amerighi, Masa Fujita, Carl Gaigné, Jon Hamilton, Jean Hindriks, Susana Peralta, Dan Sasaki, Jacques Thisse, David Wildasin, as well as seminar participants at the University of Tokyo, KIER Kyoto University, CORE Université catholique de Louvain, the HECER “Workshop on Fiscal Federalism” in Helsinki, the Workshop on “Regional Agglomeration, Growth, and Multi-level Governance” in Ghent, and the ACI Workshop “Villes et extensions des villes” in Paris, for helpful comments and suggestions. Kristian Behrens gratefully acknowledges financial support from the European Commission under the Marie Curie Fellowship MEIF-CT-2005-024266. The usual disclaimer applies.

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We develop a model of capital tax/subsidy competition in which imperfectly competitive firms choose both the number and the location of the plants they operate. The endogenous presence of horizontal multinationals is shown to attenuate the “race to the bottom” and yields some results that are opposite to traditional findings in the tax competition literature. First, in the presence of horizontal multinationals, increasing subsidies decrease firms' profits by exacerbating price competition due to more firms ‘going multinational’. Second, instead of being always subsidized, capital may actually be taxed in equilibrium. Third, taxes/subsidies become strategically independent policy instruments, instead of being strategic complements. Last, there may exist multiple equilibria with either low or high subsidies.

Keywords: capital tax competition; international trade; horizontal multinationals; foreign direct investment; imperfect competition

JEL Classification: F12; F23; H27; H73; R12
1 Introduction

National governments and local promotion agencies pervasively use targeted tax incentives, domestic infrastructure, and local skill development to attract mobile capital and to channel foreign direct investment (henceforth, FDI) flows. As a result, it is generally believed that governments’ non-cooperative behavior may get them caught in a ‘race to the bottom’: the tax game yields inefficiently low capital taxes and/or overly generous investment subsidies, with total state contribution sometimes matching up to 30% of the investments.\(^1\) This situation makes a case for international cooperation and harmonization of capital tax and investment subsidy policies. Yet, although the investment sensitivity with respect to fiscal advantages seems to be a robust empirical fact (see Cummins and Hubbard, 1995; Hines 1996), recent empirical findings mitigate the thesis of a ‘race to the bottom’ tax game between industrialized nations. Indeed, while OECD countries have observed a sharp decline in trade barriers, accompanied by even larger cross-country investment flows, during the 1980s and 90s, both effective marginal tax rates and tax revenues have remained rather stable over that period (e.g., Devereux et al., 2002). At the same time, empirical studies support the view of insignificant capital mobility between domestic and foreign locations, which suggests that multinational companies do not simply close their home production facilities to invest in foreign based ones (e.g., Devereux and Freeman, 1995).

These empirical facts qualify the traditional findings of the literature on tax competition that studies the impact of factor mobility on tax revenue and welfare. Indeed, its results generally support the idea that taxes are strategic complements, as a tax cut by one country triggers tax cuts by the others, thus giving rise to a ‘race to the bottom’ as governments are enticed to unduly cut taxes to attract the tax bases of other countries (e.g., Oates 1972; Zodrow and Mieszkowski, 1986; Wilson 1986; Wildasin 1988). Recent developments in this field, notably in the context of international trade theory and new economic geography, have analyzed the role of trade barriers and transport costs on firms’ location and tax competition (e.g., Ludema and Wooton, 2000; Baldwin and Krugman, 2004; Ottaviano and van Ypersele, 2005; Borck and Pfüger, 2006). This strand of literature explains the observed persistence of high tax rates in large regions by the presence of agglomeration rents that governments are able to tax away. Yet, by considering firms with a single production facility only, this literature overlooks the issue of FDI as a means to gain better market access, that has been emphasized in international trade theory. Be-

\(^1\)For example, total state financial incentives for the Mercedes-Benz/Swatch automobile plant in Hammbach, France, amounted to US$111 million, for a total company investment of US$370 million (Raff, 2004, p.2747). Carlton (2003) provides a detailed overview of many cases in which there is international subsidy competition between governments.
cause it assumes that firms are not allowed to alter their production structures in response to fiscal changes, this literature fails to discuss the important trade-off between market proximity and production scale that multinational companies face (Barba Navaretti and Venables, 2004). The objective of the present paper is to fill this gap by analyzing the impact of endogenous horizontal multinationals on tax competition.

In this paper, we show that governments’ competition for capital and investment does not necessarily lead to a ‘race to the bottom’ when one considers the possibility of horizontal FDI, by which firms may invest abroad in additional production facilities. As the main share of foreign investments in industrialized countries consists of horizontal FDI, the present paper lends support to the empirical evidence of stable effective tax rates and weak substitutability between home and foreign investments. To illustrate this point, we develop a two-country model of capital tax/subsidy competition in which mobile firms endogenously choose both their location and production structure. As in the paper by Ottaviano and van Ypersele (2005), all firms sell their products in both countries and face trade costs for international shipments. They hence choose to operate a single plant in one of the two countries when transport costs or trade barriers are small, whereas they build a second plant and incur additional fixed costs in the opposite case. Firms operating only a single plant are called ‘exporters’, whereas those operating multiple plants are horizontal multinationals, which we simply refer to as ‘multinationals’. Utilitarian governments offer subsidies (or tax credits) to attract firms and to increase their residents’ consumption surplus and profit claims.

We show that the outcome of the tax competition game crucially depends on the level of trade costs and the cost of capital before subsidies. First, if trade costs are low relative to the cost of capital, firms will always choose to serve both markets from a single production site. In that case, the tax base is fully mobile and firms react in the usual way to tax differences by changing locations: tax rates and investment subsidies are strategic complements and capital will always be subsidized in any non-cooperative tax equilibrium (e.g., Wilson, 1999; Haufler and Wooton, 1999; Ottaviano and van Ypersele, 2005). Second, if trade costs are high relative to the investment cost, firms will always choose

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2Baldwin and Krugman (2004), Haufler and Pflüger (2004), and Borck and Pflüger (2006) develop ‘new economic geography’ models of capital tax competition with product differentiation, trade costs, and imperfect competition. Contrary to our contribution, they do not allow for multinationals, and their CES framework abstracts from pro-competitive effects. Ottaviano and van Ypersele (2005) develop a model similar to ours featuring pro-competitive effects. Yet, they do not allow for multinationals by assuming that firms operate in a single country only. Most other contributions deal with multinationals by focussing on the location and investment choices of a single monopolist (e.g., Haufler and Wooton, 1999, 2005; Devereux and Hubbard, 2003; Raff 2004). The international trade literature dealing with multinationals usually disregards issues of tax competition and the impacts this may have on firms’ choices (e.g., Markusen and Venables, 1998, 2000; Navaretti and Venables, 2004).
to run multiple production plants and serve each market locally through subsidiary sales. By making such investments, firms gain access to spatially separated markets and save on trade and transport costs. In that case, multinationals are more profitable than exporters and governments may tax away the ‘organizational rents’ without triggering relocation. Consequently, when all firms engage in horizontal FDI, the tax bases becomes inelastic and tax competition is absent. There would, for example, be no scope for tax competition in a world where all firms are ‘McDonald-type’ multinational corporations which find it profitable to build and operate an outlet in each locale to sell their ‘hamburgers’ there.3

The most interesting case occurs for intermediate values of trade costs where multinationals and exporters coexist. Here, subsidies have an impact on firms’ organization, rather than firms’ location, since they induce exporters to build additional plants in the countries that offer large enough subsidies. In that case, higher subsidies reduce the cost of capital and increase the number of horizontal multinationals operating in the global economy. As a result, local investment subsidies have no impact on firms’ production in foreign countries, and governments’ subsidies become strategically independent. Furthermore, we show that subsidies are lower in the presence of multinationals: when compared to the case in which firms are constrained to operate a single plant only, as is usually assumed in the tax competition literature, the presence of multinationals leads to lower equilibrium subsidies.

Our analysis further reveals two important differences between firms’ location and structure in the presence and in the absence of horizontal multinationals. First, in the presence of multinationals, the location of production facilities depends on the absolute level of taxes and subsidies across countries, whereas it depends on the difference in taxes or subsidies when all firms operate a single plant only. Second, the impact of taxes and subsidies on industry profits crucially depends on the presence of multinationals. Whereas higher subsidies raise industry profits when all firms exports from a single production facility, they reduce them as soon as some firms have chosen a horizontal multinational structure by building additional plants in foreign countries. In other words, subsidies have a pro-competitive effect in the presence of multinationals. This is because an increase in the level of subsidies reduces investment costs and leads to the entry of new multinationals, which set lower markups and lower prices than exporters because they ‘jump’ trade barriers. The local industry then becomes more competitive and the foreign exporters find it more difficult to penetrate the local market. As a consequence, the remaining exporters are also enticed to invest in additional plants, which increases further the number of production sites and, therefore, product market competition. Stated dif-

3Note that this result is similar to the one obtained in models with asymmetric country sizes, in which the larger country displays a ‘home market effect’ and may usually tax away ‘agglomeration rents’ (e.g., Haufler and Wooton, 1999; Baldwin and Krugman, 2004; Ottaviano and van Ypersele, 2005).
ferently, in the presence of multinationals, subsidies have a negative effect on industry profits and, by the same token, have a stronger effect on local consumption surplus.\textsuperscript{4}

While these pro-competitive effects are good news for consumers, they are bad news for residents with large profit claims. Hence, it is important to note that subsidy or tax equilibria crucially depend on the way profits are redistributed to residents. Suppose, for instance, that profits mainly accrue to foreign shareholders, so that governments put no weight on industry profits in their objective functions. Then we show that there exists a set of parameter values that supports \textit{multiple equilibria}, one of which has \textit{low subsidies and exporters only}, whereas the other has \textit{higher subsidies and a combination of exporters and multinationals}. The existence of an equilibrium with higher subsidies and multinationals is simply explained by the fact that the fall in industry profits does not affect governments and that subsidies have larger effects on consumer surplus in the presence of multinationals. Governments may thus subsidize more. By contrast, when profits are mainly redistributed to residents, governments care about profit levels and therefore set lower subsidies to reduce the pro-competitive effects. The difference between subsidies in the presence and in the absence of multinationals is smaller, and multiple equilibria need no longer occur.

The remainder of the paper is organized as follows. In Section 2, we present the model and derive the equilibrium for given tax rates and a given spatial structure. Section 3 then discusses the spatial allocation and firms’ production structures, taking tax rates still as given. In Section 4, we describe governments’ tax choices when they play a non-cooperative game, and we fully characterize the equilibria as a function of the before-tax cost of capital. Section 5 discusses the robustness of our main results and presents some comparative statics. Section 6 concludes.

2 The model


2.1 Preferences

Consider an economy with two countries, labeled \( H \) and \( F \). Variables associated with each country will be subscripted accordingly. Each country is endowed with a mass of \( L/2 \) immobile consumers, which have identical quasi-linear preferences over a homogeneous good and a continuum of varieties of a horizontally differentiated good. The subutility

\textsuperscript{4}This feature of multinational activity goes unnoticed in models focusing on the choices of a single monopolist, as well as in models relying on CES preferences and constant mark-ups.

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over the varieties $v \in [0, 1]$ of the differentiated good is quadratic as in Ottaviano et al. (2002). The utility of a representative consumer in country $i = H, F$ is given by:

$$U_i = \sum_{j=H,F} \int_{\Omega_j} q_{ji}(v)dv - \frac{\beta - \gamma}{2} \sum_{j=H,F} \int_{\Omega_j} [q_{ji}(v)]^2 dv - \frac{\gamma}{2} \left[ \sum_{j=H,F} \int_{\Omega_j} q_{ji}(v)dv \right]^2 + q_i^0,$$

where $q_{ji}(v)$ denotes the consumption in country $i$ of the differentiated variety $v$ produced in country $j$; $\Omega_j$ stands for the set of varieties produced in country $j$, with measure $N_j$ (and with $N_H + N_F = 1$); $q_i^0$ stands for the consumption of the homogeneous good in country $i$; and $\alpha > 0$, $\beta > \gamma > 0$ are parameters.

Each consumer is endowed with one unit of labor, which she supplies inelastically, $\overline{q}_o$ units of the homogeneous good ($\overline{q}_o > 0$), and one unit of the world capital stock. Furthermore, we assume that each agent has identical claims to firms’ profits, an assumption that we relax later in Section 5. A consumer in country $i = H, F$ maximizes her utility subject to the budget constraint

$$\sum_{j=H,F} \left[ \int_{\Omega_j} p_{ji}(v)q_{ji}(v)dv \right] + p_i^0q_i^0 \leq w_i + p_i^0\overline{q}_o + r + \frac{\Pi_w}{L},$$

where $p_{ji}(v)$ is the consumer price in country $i$ of variety $v$ produced in country $j$; $w_i$ is the wage rate in country $i$; $r$ is the rental rate of a unit of capital, and $\Pi_w$ stands for world profits.

In what follows, we assume that all varieties produced in the same country are symmetric, which allows us to alleviate notation by dropping the variety index. Consumer demands are then as follows:

$$q_{ij} = a - (b + c)p_{ij} + cP_j$$

where $a$, $b$ and $c$ are positive coefficients given by

$$a \equiv \frac{\alpha}{\beta} \quad b \equiv \frac{1}{\beta} \quad c \equiv \frac{\gamma}{(\beta - \gamma)\beta}$$

and where

$$P_j \equiv N_jp_{jj} + N_ip_{ij} \quad i \neq j$$

can be interpreted as the average price of differentiated goods in country $j$.

### 2.2 Technology and trade costs

The homogeneous good is produced under perfect competition using one unit of labor only. Profit maximization in this sector implies that $p_i^0 = w_i$. We assume that this good can be
traded at no cost between countries and we choose it as the numéraire: \( p_H^* = p_F^* = 1 \). We furthermore assume that labor is intersectorally mobile, so that wages in all industries are given by \( w_H = w_F = 1 \).

Differentiated goods are produced by firms operating under increasing returns to scale. Each firm owns some firm-specific asset (e.g., a patent right) which grants it monopoly power over a single variety. The total mass of firms is, therefore, equal to the total mass of varieties, which is fixed to 1. The total mass of firms being fixed, firms make (strictly) positive profits. This is either because entrepreneurs are scare, or because the number of patents is limited, or because there exist significant barriers to entry in the industry.

Firms make both a locational and an organizational choice as they choose both the location and the number of plants they operate. Stated differently, they can choose to become either exporters (one plant) or multinationals (two plants). Denote by \( n_i \) the mass of exporters based in country \( i = H, F \) and by \( m \) the mass of multinationals, respectively.\(^5\)

Since each exporter and multinational firm produces a single variety, we must have that the number of varieties \( N_i \) is equal to \( n_i + m \) in each country \( i = H, F \). The total mass of varieties being equal to 1, we thus have

\[
N_H + N_F = n_H + n_F + m = 1.
\]

All firms have access to the same technology and produce their variety by using both labor and capital. Following Ottaviano and Thisse (2004), we assume that labor enters only the variable cost, whereas the fixed cost is incurred in terms of capital only. Without loss of generality, we may set the marginal labor requirement to zero since this amounts to rescaling firms’ demand intercepts (see Ottaviano et al., 2002). Firms require \( f \) units of capital to set up a plant in any country, and capital is assumed to be perfectly mobile across countries and sectors. We assume that the rental rate \( r \) of capital is exogenously fixed. Constant rental rates may reflect the constant degree of intertemporal substitution of lenders, or simply the fact that the industry is small when compared to the rest of the economy. Without loss of generality we normalize \( r \) to one.

When governments do not subsidize capital, the cost of a plant is equal to \( f \). When governments in \( H \) and \( F \) do subsidize capital at the rates \( \sigma_H \) and \( \sigma_F \), the costs of a plant are equal to \( f (1 - \sigma_H) \) and \( f (1 - \sigma_F) \), respectively. For the sake of exposition, we denote the firms’ after-subsidy fixed costs by \( f - s_H \) and \( f - s_F \), where \( s_H \equiv f \sigma_H \) and \( s_F \equiv f \sigma_F \) stand for the values of subsidies. Hence, the subsidies \( s_H \) and \( s_F \) are lump sum transfers to the firms, which depend on the number and location of their investments. Note that all

\(^5\)We do not need to keep track of where multi-plant firms are headquartered. This is because we assume that fixed costs are the same in both countries. When fixed costs differ, headquarters will be exclusively located in the low fixed cost country (Navarette and Venables, 2004, p.54).
firms in a country are equally subsidized.\textsuperscript{6} Subsidies to local exporters can be interpreted as incentives for firms not to relocate their plants abroad. Thus, subsidies play the dual role of trying to attract new firms and preventing existing firms from leaving. Note, finally, that nothing precludes subsidies a priori from being negative, in which case they are equivalent to source-based capital taxes payed in the location where the capital is used.\textsuperscript{7}

 Turning to transportation, shipping each variety of the differentiated good across countries is costly, whereas shipping it within each country is free. More specifically, shipping one unit of any variety between the two countries entails a per-unit cost of $\tau > 0$ units of the numéraire. Note that the existence of transport costs and, since we focus on capital taxation, the absence of transfer pricing problems, ensure that multinationals behave like local firms in each domestic market and serve that market through local sales only. Indeed, given plant-level scale economies and transport costs the firm will never produce a fraction of demand locally while importing the rest from abroad. Hence, our model is of the ‘proximity-vs-scale’ type (Barba Navaretti and Venables, 2004).\textsuperscript{8}

 Firms and governments play a three-stage game. In the first stage, governments set non-cooperatively their taxes (subsidies) to firms. In the second stage, firms choose the number and location of their production plants. Finally, in the third stage, firms set profit maximizing prices given the previous choices. We solve the game by backward induction for its subgame-perfect Nash equilibria.

 In the following sub-section, we present the market outcome. We then discuss firms' locational and organizational choices in Section 3, and the tax equilibria in Section 4.

\subsection*{2.3 Market outcome}

Since our framework features a continuum of firms, each firm is negligible to the market and sets its own prices taking all other variables as given. Considering $n_H$, $n_F$ and $m$, as well as the subsidies $s_H$ and $s_F$, as fixed, firms maximize their profits with respect to

\textsuperscript{6}Such equal treatment of firms circumvents the problem of tax discrimination that arise when tax incentives apply only to non-residents (UNCTAD, 2000). For example, “the EU adopted a Code of Conduct for business taxation, in which member states committed themselves to refrain from ‘unfair’ tax policies” (Haufler and Wooton, 2005, p.2).

\textsuperscript{7}In this paper, we do not consider residence-based taxes. Under perfect information, residence-based taxes are non-distortionary lump sum transfers which do not generate tax competition between governments. The main discussion about residence-based taxes arises under imperfect information when tax authorities can hardly observe and collect the foreign capital incomes of their residents. This discussion is beyond the scope of the present paper. Furthermore, Keen (1993) argues that the effective taxation of multinationals is source based, independently of what tax codes effectively stipulate.

\textsuperscript{8}When all firms are horizontal multinationals, there is no more trade in our model. This is a strong result that may not hold when multinationals are also multiproduct firms (Baldwin and Ottaviano, 2001).
prices. In accord with empirical evidence (see, e.g., Head and Mayer, 2000; Haskel and Wolf, 2001), we assume that international markets are segmented, i.e., firms are free to set prices specific to each national market they sell their product in.

In what follows, we superscript variables pertaining to exporters and to multinationals by \( x \) and by \( m \), respectively. The profit before subsidy of an exporter established in country \( i = H, F \) is given by

\[
\Pi^x_i = \frac{L}{2} p_{ii} q_{ii} + \frac{L}{2} (p_{ij} - \tau) q_{ij} - f,
\]

whereas the profit before subsidy of a multinational is given by

\[
\Pi^m = \frac{L}{2} p_{HH} q_{HH} + \frac{L}{2} p_{FF} q_{FF} - 2f.
\]

Since multinationals serve each market locally only, their local pricing decisions are identical to those of domestic firms operating in same market. Firms maximize their profits with respect to own prices taking the average prices \( P_H \) and \( P_F \) as given. Substituting firms’ optimal prices in the average prices \( P_i \) and solving for the price equilibrium yields:

\[
\begin{align*}
p^*_i &= \frac{2a + cn_j \tau}{2(2b + c)} \quad p^*_j = p^*_i + \frac{\tau}{2} \\
q^*_i &= (b + c) p^*_i \quad q^*_j = (b + c)(p^*_j - \tau),
\end{align*}
\]

for \( i = H, F \) and \( j \neq i \). Because \( n_H + n_F + m = 1 \), the equilibrium price in country \( H \) can also be expressed as follows:

\[
p^*_{HH} = \frac{2a + c\tau - c(m + n_H)\tau}{2(2b + c)}.
\]

This reveals that prices in country \( H \) decrease with the mass of plants located in that country \( (m + n_H) \). Note that the mass of multinationals appears in both countries' equilibrium prices. Hence, whereas exporters push prices down only in the country they locate in, multinationals put downward pressure on prices in both countries, and the more so the higher trade costs are. As we will show below, this property has important consequences on the equilibrium outcome of the tax game.

To simplify the analysis and to avoid a proliferation of sub-cases, we assume that trade costs are sufficiently low such that international trade is always feasible. It is readily verified that \( q^*_{HF} > 0 \) and \( q^*_{FH} > 0 \) for all allocations \((n_H, n_F, m)\) provided that the following trade feasibility condition holds:

\[
\tau < \frac{2a}{2b + c}.
\]

Under Condition (3), international trade occurs regardless of government’s subsidies and firms’ location and organization.
Using the profit-maximizing prices (1) and quantities (2), the profits can be expressed as follows:

\[ \Pi_i^x = \frac{L(b+c)}{2} \left[ (p_{ii}^*)^2 + \left( p_{jj}^* - \tau \right)^2 \right] - f \]  
\[ \Pi_m^x = \frac{L(b+c)}{2} \left[ (p_{HH}^*)^2 + (p_{FF}^*)^2 \right] - 2f. \]

Finally, the consumer surplus of a resident in country \( i = H, F \) is given by

\[ S_i = \frac{a^2}{2b} - a \left[ (n_i + m) p_{ii}^* + n_j p_{ji}^* \right] + \frac{b+c}{2} \left[ (n_i + m) (p_{ii}^*)^2 + n_j (p_{ji}^*)^2 \right] - \frac{c}{2} \left[ (n_i + m) p_{ii}^* + n_j p_{ji}^* \right]^2. \]

We now turn to the analysis of firms’ locational and organizational choices.

3 Firms’ structure, location, and profits

Subsidies affect firms’ location, production structure, and profits. We begin by investigating which types of equilibrium configurations \((n_H^*, n_F^*, m^*)\) may arise for any given couple of subsidies \((s_H, s_F)\). We then discuss how subsidies affect firms’ profits in the different configurations.

3.1 Structure and location

Only two equilibrium configurations are relevant for the subsidy game between governments. For the sake of conciseness, we therefore only discuss these configurations in detail.

(i) A pure exporter configuration includes only exporters (i.e., \( m^* = 0 \)). In this case, subsidy-inclusive profits of exporters must be equalized across countries. Evaluating (4) and (5) at the equilibrium prices (1), the profit differential between exporters in countries \( H \) and \( F \) is equal to

\[ \Pi_H^x + s_H - (\Pi_F^x + s_F) = K (n_F - n_H) + s_H - s_F, \quad K \equiv \frac{c(b+c)}{4(2b+c)} L \tau^2 > 0. \]

Equating this profit differential to zero, the equilibrium masses of exporters and plants in countries \( H \) and \( F \) are given by

\[ n_H^* = \frac{1}{2} + \frac{1}{2K} (s_H - s_F) \quad \text{and} \quad n_F^* = \frac{1}{2} - \frac{1}{2K} (s_H - s_F). \]

Hence, by making capital cheaper, a country’s subsidy attracts exporters from the other country. In a pure exporter configuration, subsidy competition is a zero-sum game in terms of number of local firms and, therefore, in term of product access for consumers.
Feasibility of a pure exporter configuration further requires that exporters are more profitable than multinationals, and that $n^*_H$ and $n^*_F$ are non-negative:

$$\Pi^m_i + s_j - \Pi^x_i \leq 0, \quad i = H, F \quad \iff \quad s_H + s_F \leq 2B - K \quad (8)$$

$$n^*_H \geq 0 \quad \iff \quad s_F \leq s_H + K \quad (9)$$

$$n^*_F \geq 0 \quad \iff \quad s_F \geq s_H - K, \quad (10)$$

where all profits are computed using the equilibrium prices (1), evaluated at $n_H = n^*_H$, $n_F = n^*_F$ and $m = 0$, and where

$$B \equiv f - \frac{(b + c)[4a - (2b + c)\tau]}{8(2b + c)}L\tau.$$ 

The set of subsidies supporting this configuration, delimited by conditions (8)–(10), is depicted by zone (i) in Figure 1.

(ii) A mixed configuration occurs when exporters and multinationals coexist (i.e., $m^* > 0$, $n^*_H \geq 0$ and $n^*_F \geq 0$). Therefore, subsidy-inclusive profits of all types of firms must be equalized across countries: $\Pi^*_H + s_H = \Pi^*_F + s_F = \Pi^m + s_H + s_F$. Note that this configuration includes the particular case where all firms are multinationals, but are just indifferent between building one or two plants.

The profit differential between an exporter in country $H$ and a multinational can be expressed as follows:

$$\Pi^m + s_F - \Pi^x_H = \frac{L}{2} (b + c) \tau \left( p^*_{FF} - \frac{\tau}{4} \right) - f + s_F,$$

a symmetric expression holding for country $F$. Equating the above profit differentials to zero, we readily obtain the following equilibrium masses of plants in countries $H$ and $F$:

$$n^*_H + m^* = \frac{K - 2B + s_H}{K} \quad \text{and} \quad n^*_F + m^* = \frac{K - 2B + s_F}{K}.$$ 

By reducing the net cost of capital, a country’s subsidy increases the number of plants in its market. Since $n_H + n_F + m = 1$, we get the equivalent conditions

$$n^*_H = \frac{B - s_F}{K}, \quad n^*_F = \frac{B - s_H}{K}, \quad m^* = \frac{K - 2B + s_H + s_F}{K}. \quad (11)$$

Before going further, it is important to observe the following two points. First, the mass of exporters $n^*_H$ in country $H$ is independent of the subsidy $s_H$ set by this country,

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9The parameter values underlying Figure 1 are as follows: $\alpha = \beta = 1$, $\gamma = 0.5$, $L = 4$, $\tau = 0.1$, and $f = 3/40$. 

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because it only induces exporters in country $F$ to build a second plant in country $H$. Second, an identical increase in subsidy has a stronger impact on the mass of local plants and varieties in the mixed configuration than in the pure exporter configuration. Indeed, comparing (11) and (7), respectively, one can check that $\partial (n_H^*+m^*)/\partial s_H = 1/K$ is larger than $\partial n_H^*/\partial s_H = 1/(2K)$. The intuition is that, by ‘going multinational’, firms make the global economy more competitive, thereby cutting profits of single-plant exporters by making their access to foreign markets more difficult. This in turn entices even more exporters to ‘go multinational’, thereby further increasing competition. As a result, the effect of a subsidy on the number of local plant is stronger in the mixed configuration.

Finally, feasibility of the configuration requires that

$$n_H^* \geq 0 \iff s_F \leq B$$

$$n_F^* \geq 0 \iff s_H \leq B$$

$$m^* > 0 \iff s_H + s_F > 2B - K.$$  

Condition (14) is simply the counterpart of condition (8). The set of subsidies supporting this configuration, delimited by conditions (12)–(14), is depicted by zone (ii) in Figure 1.

We summarize our results in the following proposition:

**Proposition 1 (firms’ location)** In a pure exporter configuration where each firm exports from a single plant, only the subsidy differential has an effect on the location of production. In a mixed configuration where exporters coexist with multinationals, subsidies have an effect on both organization and location structures of firms. A higher level of subsidy in a country entices some exporters to build up of a second plant there and has no impact on the production in other country.

As shown in Figure 1 and in Appendix 1, other configurations exist. For instance, when subsidies are very large, there exists a configuration (iii) that includes only multinationals. When the subsidy $s_H$ is sufficiently larger (resp smaller) than the subsidy $s_F$, there exist configurations (iv) and (v) (resp., (iv’) and (v’)) which include no exporters in country $F$ (resp., in country $H$). Yet, as shown in Section 4, such configurations are never equilibria of the subsidy game between governments, so that we may disregard them.

### 3.2 Profits

Because subsidies affect firms’ structure and location, their impact on subsidy-inclusive profits is a priori ambiguous. A more careful analysis of this point will prove useful in guiding intuition on the impact of multinationals on subsidy equilibria.

In the pure exporter configuration (i), a subsidy increase in a country directly raises the profit of firms located in that country. Moreover, since the subsidy increase entices firms
to leave the other country, and thus weakens competition there, it also raises the profits of firms in the other country. Indeed, in (the interior of the domain of) configuration (i), we can show that
\[
\frac{d (\Pi_H^x + s_H)}{ds_H} = \frac{b + n_H^* c}{2b + c} > 0, \quad \frac{d (\Pi_H^x + s_H)}{ds_F} = \frac{b + n_F^* c}{2b + c} > 0.
\]

Yet, this result does not hold in the presence of multinationals, since subsidies affect the number of plants rather than firms’ location. In fact, a subsidy increases competition in the country where new plants are built, whereas it leaves competition in the other country unchanged. Thus, competition in the global economy increases and profits fall in both countries. In configuration (ii), we can show that
\[
\frac{d (\Pi_H^x + s_H)}{ds_H} = \left(1 - \frac{2p_{HH}^*}{\tau}\right) < 0, \quad \frac{d (\Pi_H^x + s_H)}{ds_F} = \left(1 - \frac{2p_{FF}^*}{\tau}\right) < 0,
\]

since \(p_{HH}^* > \tau/2\) and \(p_{FF}^* > \tau/2\) under the trade feasibility condition (3). Hence, the impact of subsidies on profits is drastically modified in the presence of multinationals, due to the possibility of firms’ endogenously changing their production structure.

**Proposition 2 (firms’ profits)** Suppose that governments increase their subsidies (reduce their taxes). Then, profits after subsidy increases in a pure exporter configuration (i) whereas they fall in a mixed configuration (ii).

It is easy to show that profits are minimized in two cases. Firstly, when firms are heavily taxed (i.e., \((s_H, s_F) \rightarrow (-\infty, -\infty)\) in a pure exporter configuration, which yields negative profits below some threshold); and secondly, when firms are highly subsidized in the mixed configurations (i.e., \((s_H, s_F) = (B, B)\), which yields a local minimum for profits). In what follows, we restrict our attention to the meaningful situation in which firms’ subsidy-inclusive profits are positive so that production takes place. This imposes, firstly, that taxes are not too high so that firms make positive profits in configuration (i); and, secondly, that profits are positive at the point \((s_H, s_F) = (B, B)\), which implies that firms’ profits are always positive for any optimal subsidies in configuration (ii). Because \(\Pi H^* + s_H + s_F = \Pi F^* + 2B\) when \(s_H = s_F = B\), the formal condition for this is that
\[
\frac{1}{4} L (b + c) \left(\tau^{\text{trade}} - \tau\right)^2 > 0,
\]

which is always satisfied since \(\tau < \tau^{\text{trade}}\).

To sum up, our analysis reveals that subsidies have a strong impact on firms’ location, structure, and profits. In particular, when subsidies are small relative to fixed costs, the industry consists of exporters only, whereas it also includes multinationals when they are larger. In the absence of multinational firms, the location of production is quite sensitive...
to the difference in subsidies. By contrast, in the presence of multinational firms, the difference in subsidies plays no role in the sense that a government does not lose local production when the other country increases its subsidy. Finally, firms may gain or lose from higher subsidies, depending on their organizational structure. In particular, profits \textit{may well decrease with larger subsidies as the global economy becomes more competitive due to an increasing mass of multinationals}. We now explore the impact of the existence of horizontal multinationals on governments’ subsidy competition.

4 Subsidy competition

Ottaviano and van Ypersele (2005) have shown that competing governments subsidize capital to attract mobile single-plant firms. What happens when firms can choose both the location and the number of plants? How does the existence of multinationals affect the traditional insights gained from tax competition models?

In this section, we assume that firms and capital are wholly owned by residents of countries $H$ and $F$ (‘fully diversified portfolio’ assumption). More precisely, country $H$’s residents get one-half of firms’ profits after subsidy, and country $F$’s residents get the other half.\textsuperscript{10} The government of country $H$ chooses $s_H$ to maximize its residents’ welfare given by

\[
W_H = \frac{L}{2} S_H - (n_H + m) s_H + \frac{1}{2} \left[ n_H (\Pi_H^x + s_H) + n_F (\Pi_F^x + s_F) \right.
\]
\[
+ m (\Pi_H^m + s_H + s_F) \left. + \frac{L}{2} q_0 + \frac{K}{2} \right],
\]

which consists of local consumer surplus minus the direct cost of subsidizing capital, plus one-half of firms’ profits, $\Pi_H^x + s_i$ and $\Pi_H^m + s_H + s_F$, plus a constant endowment and capital income term.\textsuperscript{11} Note that the local consumer surplus $S_H$ is a function of $(p_{HH}, n_H, n_F, m)$ and that profits $\Pi_H^x$ and $\Pi_H^m$ are functions of $(p_{HH}, p_{FF})$.

Differentiating $W_H$ with respect to $s_H$, we can express the impact of a marginal increase

\textsuperscript{10}Ottaviano and van Ypersele (2005) use a similar assumption. Still, it is known that the ownership structure of firms and the distribution of profits is not innocuous (e.g., Raff, 2004). In Section 5, we discuss the robustness of our results under alternative assumptions on the distribution of profits.

\textsuperscript{11}In this model, capital lenders earn a fixed income ($r = 1$) that does not depend on the spatial distribution and the organization of firms. Therefore, local welfare $W_H$ does not explicitly depend on fixed costs $f$, although it implicitly depends on them through the equilibrium location of firms.
in subsidy on local welfare as follows:

\[
\begin{align*}
\frac{dW_H}{ds_H} &= \frac{1}{2} \frac{\partial n_H}{\partial s_H} \left( L \frac{\partial S_H}{\partial n_H} + \Pi^*_H - s_H \right) + \frac{1}{2} \frac{\partial n_F}{\partial s_H} \left( L \frac{\partial S_H}{\partial n_F} + \Pi^*_F + s_F \right) \\
+ &\frac{1}{2} \frac{\partial m}{\partial s_H} \left( L \frac{\partial S_H}{\partial m} + \Pi^m - s_H + s_F \right) \\
+ &\frac{1}{2} \frac{\partial \Pi^*_H}{\partial \partial S_H} \left( n_H \frac{\partial \Pi^*_H}{\partial \partial S_H} + n_F \frac{\partial \Pi^*_F}{\partial \partial S_H} + m \frac{\partial \Pi^m}{\partial \partial S_H} \right) \\
+ &\frac{1}{2} \frac{\partial \Pi^*_F}{\partial \partial S_H} \left( n_H \frac{\partial \Pi^*_H}{\partial \partial S_H} + n_F \frac{\partial \Pi^*_F}{\partial \partial S_H} + m \frac{\partial \Pi^m}{\partial \partial S_H} \right) - \frac{1}{2} (n_H + m).
\end{align*}
\]  

(16)

In this expression, the last term captures the direct cost of a subsidy paid to all firms. The first three terms represent the indirect effects of the subsidy through relocation of firms at constant prices; whereas the intermediate terms finally capture the indirect effects of a subsidy through price changes.

### 4.1 Governments’ best responses

We now determine the subsidies government \( H \) may potentially choose in the subsidy game for a given value of \( s_F \). This allows us then to derive and discuss governments’ best responses associated with different spatial structures. All standard calculations are relegated to Appendix 2.

To begin with, observe that government \( H \) will always choose subsidies that yield either configuration (i) or (ii). This observation justifies our focus on these two configurations in previous sections. Assume, indeed, that the subsidy \( s_H \) were sufficiently large so that country \( F \) would include no exporters, which corresponds to either configuration (iii), (iv) or (v) in Figure 1. Then every firm would operate a plant in country \( H \) and possibly one in country \( F \). If government \( H \) increased its subsidy, this would not alter the distribution of exporters across the two countries. Furthermore, because the subsidy \( s_H \) has no impact on the decision to build a plant in country \( F \), this action would not alter firms’ organizational structure. Hence, the only effect of such a subsidy would be to raise firms’ profits, which cannot be welfare improving since some profits accrue to foreign shareholders.\(^{12}\)

Formally, one can check in configurations (iii), (iv) and (v) that \( \frac{\partial n_H^*}{\partial s_H} = \frac{\partial n_F^*}{\partial s_H} = \frac{\partial m}{\partial s_H} = 0 \), so that marginal welfare (16) reduces to

\[
\frac{dW_H}{ds_H} = -\frac{1}{2} (n_H^* + m^*) = -\frac{1}{2} < 0.
\]  

(17)

By consequence, government \( H \) will never set a subsidy compatible with configurations (iii), (iv) and (v); whereas, by symmetry, government \( F \) will never set a subsidy compatible

\(^{12}\)As shown in Section 5, this result holds for all possible ways of distributing profits between domestic and foreign shareholders and, therefore, does not depend on our assumption of an equal redistribution.
with configurations (iii), (iv’) and (v’). Tax competition equilibria therefore necessarily occur in configurations (i) and (ii) (including their borders), which we now analyze in more detail.

We start by determining government $H$’s best response functions for subsidies supporting configurations (i) and (ii) separately. We then combine the pieces of these two best response functions and derive the equilibria of the subsidy game.

**Pure exporter configurations (i):** Suppose that the industry consists of only exporters, as in configuration (i) above. Straightforward calculation shows that changes in prices and locations are given by

$$\frac{\partial p_{HH}}{\partial n_F} = \frac{c\tau}{2(2b+c)} \quad \text{and} \quad \frac{\partial n_H^*}{\partial s_H} = -\frac{\partial n_F^*}{\partial s_H} = \frac{1}{2K}.$$ 

The marginal welfare (16) can hence be rewritten as follows:

$$\frac{dW_H}{ds_H} = -\frac{1}{2} n_H - \frac{\partial n_H}{\partial s_H} s_H + \frac{\partial n_H}{\partial s_H} L \left( \frac{\partial S_H}{\partial n_H} - \frac{\partial S_H}{\partial n_F} \right) + \frac{1}{2} \frac{\partial p_{HH}}{\partial n_H} \frac{\partial p_{HH}}{\partial n_F} \left[ L \frac{\partial S_H}{\partial p_{HH}} + n_H \frac{\partial \Pi_H^e}{\partial p_{HH}} + n_F \frac{\partial \Pi_F^e}{\partial p_{HH}} \right] + \frac{1}{2} \frac{\partial p_{FF}}{\partial n_H} \frac{\partial p_{FF}}{\partial n_F} \left( n_H \frac{\partial \Pi_H^e}{\partial p_{FF}} + n_F \frac{\partial \Pi_F^e}{\partial p_{FF}} \right).$$

(18)

The first term in (18) stands for the cost of the subsidy paid to all firms located in country $H$. Indeed, when government $H$ increases its subsidy $s_H$ it pays an additional amount to all firms established in $H$, with one-half of this subsidy being recouped by local shareholders through profit redistribution. The second term captures the cost of the subsidy due to firms relocating to country $H$. Since profits across countries are equal in equilibrium, shareholders are unaffected by such a relocation and the cost of the subsidy is simply equal to $s_H$. The third term captures the impact of firms’ relocation on local consumer surplus. An increase in the subsidy $s_H$ attracts plants to country $H$, so that more varieties are produced and sold locally at the price $p_{HH}$ which allows consumers to save on transport costs. The fourth term is identically equal to zero. This is because when profits are evenly redistributed across countries in the model by Ottaviano et al. (2002), an increase in $p_{HH}$ reduces the consumer surplus of home residents and augments their share in total profits in exactly the same proportions. The last term finally captures the effect of a change in foreign prices due to the subsidy. An increase in $s_H$ entices firms to leave country $F$, which increases prices and profits there and thus benefits to local shareholders. To sum up, the government balances the costs of the subsidy and its associated profit effects with the benefits in local consumer surplus and foreign profits.

In Appendix 2, we show that $dW_H/ds_H$ is a decreasing function of $s_H$ and that
\[ dW_H/ds_H = 0 \text{ if and only if} \]
\[ s_H = \tilde{s}_H(s_F) = \frac{8b + 3c}{16b + 7c} s_F 
+ \tau L (c + b) \frac{4a (b + c) - \tau (2b^2 + 4bc + c^2)}{2 (2b + c) (16b + 7c)}, \]  

which is an affine function of \( s_F \), with positive intercept and a positive slope less than 1. When government \( F \) increases its subsidy \( s_F \), government \( H \) responds by raising its subsidy \( s_H \) but by less than the full amount. Hence, in a pure exporter configuration, subsidies are \textit{strategic complements}, which is the standard result of the literature. Note that \( \tilde{s}_H \) shifts upwards when the demand for the differentiated good (i.e., \( a \) or \( L \)) increases. This is because in such a case there is a larger volume of imports so that government \( H \) has more incentives to attract firms in order to save transport costs incurred by its residents.

The graph of the best response \( \tilde{s}_H(s_F) \) is depicted in Figure 2, panels (a) and (b).\textsuperscript{13}

Note that when the graph of \( \tilde{s}_H(s_F) \) lies above the domain of configuration (i), the best response is given by the upper border (9) of the configuration. Indeed, in such a case, government \( H \) has an incentive to raise its subsidy when the latter lies in configuration (i) and, by (17), it has an incentive to reduce its subsidy when the latter lies in configuration (v). The optimal subsidy lies on the upper border (9) of configuration (i). Similarly, when the graph of \( \tilde{s}_H(s_F) \) lies below the domain of configuration (i), the best response is given by the lower border (10) of the configuration.

\textbf{Insert Figure 2 about here.}

\textbf{Mixed configurations (ii):} Suppose that the industry comprises all types of firms as is the case in the mixed configuration (ii). Expression (11) shows that changes in location are given by

\[ \frac{\partial n^*_H}{\partial s_H} = 0 \quad \text{and} \quad \frac{\partial n^*_F}{\partial s_H} = \frac{\partial m^*}{\partial s_H} = -\frac{4(2b + c)}{L (b + c) \tau^2 c} < 0. \]

Hence, an increase in country \( H \)'s subsidy has as sole effect to entice exporters in country \( F \) to become multinationals. Stated differently, the subsidy does not alter the number of plants and, therefore, the competition in country \( F \). Still, in contrast to Ottaviano and van Ypersele (2005), the subsidy \textit{affects the number of varieties produced locally in}

\textsuperscript{13}Figure 2 is drawn for the following parameter values: \( \alpha = \beta = 1, \gamma = 1/2, L = 7, \tau = 1/2, \) and \( f = 1.2 \). Panel (a) depicts government \( H \)'s best response of Section 4 (full and equal redistribution of profits to local shareholders, \( \lambda_H = \lambda_F = 1/2 \)); whereas panel (b) depicts government \( H \)'s best response of Section 5.2 (full redistribution of profits to absentee shareholder, \( \lambda_H = \lambda_F = 0 \)).
the subsidizing country, since it attracts plants. In other words, tax competition is not a
‘zero-sum game’ in terms of locally produced varieties.\textsuperscript{14}

The marginal welfare (16) can be expressed as follows:

$$\frac{dW_H}{ds_H} = \frac{1}{2} (n_H + m) + s_H \frac{\partial n_F}{\partial s_H} - \frac{1}{2} \frac{\partial n_F}{\partial s_H} \left( L \frac{\partial S_H}{\partial m} - L \frac{\partial S_H}{\partial n_F} \right)$$

$$+ \frac{1}{2} \frac{\partial p_{HH}}{\partial n_F} \frac{\partial n_F}{\partial s_H} \left( L \frac{\partial S_H}{\partial p_{HH}} + n_H \frac{\partial \Pi_H}{\partial p_{HH}} + n_F \frac{\partial \Pi_F}{\partial p_{HH}} + m \frac{\partial \Pi_H}{\partial p_{HH}} \right).$$

(20)

As in a pure exporter configuration, the first term of (20) stands for the cost of the
subsidy paid to all firms located in country $H$, when one-half of the subsidy is recouped
by local shareholders. The second term captures again the cost of the subsidy paid to the
exporters that open a second plant in country $H$. The third term captures the impact
of firms’ relocation on local consumer surplus. As new plants are built in country $H$,
competition increases, prices drop, and consumer surplus rises in that country. Finally,
the fourth term is identically equal to zero, as in the pure exporter configuration, and for
the same reasons.

In Appendix 2, we show that $dW_H/ds_H$ is a decreasing function of $s_H$ and that
$dW_H/ds_H = 0$ in a mixed configuration if and only if

$$s_H = \tilde{s} \equiv \frac{1}{2} f - \frac{1}{32} L \tau \frac{b + c}{2b + c} (4a - 2\tau b + \tau c).$$

(21)

Therefore, the subsidy that maximizes local welfare in country $H$ is independent of the
other country’s subsidy $s_F$. In the presence of multinationals, each government disregards
the action of its rival when it sets its own subsidy and tax competition may entirely
disappear.

The graph of the best response $\tilde{s}$ is also depicted in panels (a) and (b) of Figure 2.
Note that, as is the case of $\hat{s}_H$ discussed before, when the graph of $\tilde{s}$ lies above (resp.,
below) the domain of configuration (ii), the best response is given by the upper border
(13) (resp., lower border (14)) supporting this configuration.

**Switch between configurations (i) and (ii):** For some sets of parameter values, the
best response functions may have an upward jump between configurations (i) and (ii)
while they may have a continuous transition between these two configurations for other
sets of parameter values. In the latter case, the best response function is downward
sloping, thus implying that subsidies are strategic substitutes over some range. This is
shown in panel (a) of Figure 2. The existence of strategic substitutes crucially hinges on

\textsuperscript{14}This effect seems to be supported empirically. Indeed, Devereux and Freeman (1995) have reported
that tax policy hardly affects the investor’s choice between domestic and foreign investment. Even when
tax policy affects this choice, it often does not take the form of a simple relocation but consists in the
opening of an additional production unit.
governments’ trade-off between local consumer surplus and the share of producer surplus accruing to local residents. When subsidies are strategic substitutes, a rise in country $F$’s subsidy raises the number of plants there, without altering the number of plants in country $H$. Hence, whereas residents in country $H$ have access to the same number of varieties at the same price (and, therefore, enjoy the same consumer surplus), they earn less because total profits decrease with larger subsidies in the presence of multinationals (see Proposition 2). Government $H$’s best response is then to cut its own subsidy in response to the other country’s subsidy increase. By so doing, it benefits from two effects: first, it saves on costly subsidies, and second it increases profits that accrue to its residents. At the margin, these two effects outweigh the negative impact that the subsidy has on the number of plants and varieties produced in country $H$. Note, finally, that subsidies are more likely to become strategic substitutes over some range when residents are entitled to larger shares of total profits. This point will be further developed in Section 5.

Let us summarize the foregoing results in the following proposition.

**Proposition 3 (subsidy competition)** Whereas subsidies are strategic complements in pure exporter configurations, they are strategically independent in mixed configurations where multinationals and exporters coexist. On the border between the two configurations, subsidies may be strategic substitutes.

In the previous paragraphs, we have described the pieces of the best response functions for configuration (i), (ii), and the border between them. Yet, each government may set subsidies that induce different configurations, so that its best response to the other country’s subsidy may imply changes of firms’ production structures. Put differently, governments’ best response functions will consist of the combination of the pieces derived above and will include transitions between configurations, which may be continuous or discontinuous, depending on parameter values.

### 4.2 Subsidy equilibria

We now characterize the equilibrium configurations when governments simultaneously and non-cooperatively set their subsidies. Note from the outset that the existence of pure-strategy Nash equilibria in this subsidy game is guaranteed, because subsidies are defined over a compact subset of $\mathbb{R}$ and because governments’ subsidy reaction functions have no downward jump (see Vives, 1999, p.41). We first present the possible equilibrium configurations and then discuss the conditions under which they occur. As suggested by the analysis of the best response functions, subsidy equilibria may yield a configuration with only exporters, a mixed configuration including both organizational structures, or a configuration with only multinationals. We start by describing the former.
(i) A pure exporter equilibrium exhibits the traditional features of tax competition. When the tax base is internationally mobile, subsidies are strategic complements since each government raises its subsidy in reaction to an increase in the other country’s subsidy, to counteract the relocation of mobile firms. Such ‘subsidy competition’ may lead to an excessive inflation of subsidies or, equivalently, excessively low taxes. Formally, the equilibrium subsidies are such that $s^*_H = s_H(s^*_F)$ and $s^*_F = s_F(s^*_H)$, which yields

$$s^*_H = s^*_F = \tau L \left( b + c \right) \frac{4a(b + c) - (2b^2 + 4bc + c^2) \tau}{8(2b + c)^2} > 0.$$  \((22)\)

First, as can be seen from expression \((22)\), equilibrium subsidies increase with demand \((a \text{ and } L)\). When demand is large, both governments have incentives to attract firms so that local consumers save on transport costs. Second, the equilibrium subsidies are always positive. Indeed, as usual in the tax competition literature, governments impose a negative externality on each other and offer too large subsidies or set too low tax rates.\(^{15}\) Finally, subsidies are independent of firms’ fixed costs \(f\). In a pure exporter configuration, firms have to pay the fixed capital cost wherever they locate, so that governments may neglect this aspect when deciding on their optimal subsidies.

(ii) A mixed equilibrium is characterized by the following equilibrium subsidies:

$$s^*_H = s^*_F = \tilde{s},$$  \((23)\)

where \(\tilde{s}\) is given by \((21)\). This illustrates the impact on tax competition of firms being able to endogenously choose their production structure. When exporters and multinationals co-exist, governments’ subsidy choices are independent. Contrary to the pure exporter case, the equilibrium subsidies now decrease with larger demand \((\text{i.e., } a \text{ and } L)\), whereas they increase with respect to the cost of a plant. The latter effect is due to the fact that firms may now choose not to pay the cost of the second plant when it is too high; whereas the former effect stems from the fact that increasing demands raise operating profits of multinationals (and therefore, in equilibrium, of all firms), which allows governments to subsidize less (resp., to tax more) without triggering a structural change in firms’ organization.

As a limit case of the mixed equilibrium (ii), the subsidy game may yield a pure multinational equilibrium. This corresponds to the case where the graph of \(\tilde{s}\) lies above the domain of configuration (ii), and where the best response function thus lies on the border between configurations (ii) and (iv) where only multinationals survive. This case naturally occurs for small fixed costs \(f\) of a plant. On the one hand, when the economy consists of multinationals only, as in configuration (iii), these firms constitute an immobile

\(^{15}\)See Ottaviano and van Ypersele (2005) for a welfare analysis of the pure exporter configuration.
tax base which governments tend to tax as much as possible without triggering organizational changes. Hence, the subsidy equilibrium lies at the point at which any further decrease in subsidies entices some multinationals to modify their structure. On the other hand, a lower subsidy leading to configuration (ii) cannot improve local welfare as its costs for local consumers are higher than its benefits. Therefore, subsidy competition yields an equilibrium where all firms own two plants but are almost willing to shut one down. In such a situation, governments set equilibrium subsidies equal to

$$s^*_H = s^*_F = B = f - \frac{(b + c)[4a - (2b + c)\tau]}{8(2b + c)}L\tau. \quad (24)$$

Subsidies are thus decreasing in demands (i.e., $a$ and $L$) and in transport costs (i.e., $\tau$), for the same reasons as in the mixed configurations (ii). Note, finally, that the equilibrium subsidies increase one-to-one with respect to $f$. This is because any increase in $f$ decreases multinationals’ profits by exactly that amount, which must be offset by an increase in subsidies for firms to remain multinational in equilibrium.

Finally, the subsidy game may yield a continuum of pure exporter equilibria that lie on the downward sloping part of the best response function. The associated equilibrium subsidies are such that all firms choose to export but are almost willing to become multinational. The equilibrium subsidies satisfy the following relationship:

$$s^*_H + s^*_F = 2B - K = 2f - \left(a - \frac{\tau b}{2}\right)L\tau\frac{b + c}{2b + c}. \quad (25)$$

and are decreasing in the volume of demand and in transport costs. It is noteworthy that, in this configuration, we have asymmetric equilibria in an otherwise perfectly symmetric set up. Governments may choose not to set the same subsidies, and there is a continuum of ways of doing so. In such an equilibrium, each government raises its subsidy up to the level that triggers the entry of multinationals. Indeed, below this level, each government competes with the other to attract exporters and to allow its consumers to save on transport costs. Above this level, the additional competition induced by multinationals deteriorates profits so much that the increase in subsidy reduces local profit earnings more than it increases consumer surplus.

We now establish the precise conditions under which the above equilibria occur. In Appendix 3, we define three thresholds $0 < f_1 < f_2 < f_3$. Using those thresholds, we prove the following proposition:

**Proposition 4 (subsidy equilibria)** The equilibrium of the subsidy game yields

- a pure multinational configuration if and only if $f \leq f_1$;
• a mixed configuration with multinationals and exporters if and only if $f \in (f_1, f_2)$;
• a continuum of pure exporter configurations if and only if $f \in [f_2, f_3)$;
• a pure exporter configuration if and only if $f \geq f_3$.

**Proof.** See Appendix 3. ■

As expected, pure multinational equilibria of the tax game occur when the fixed cost $f$ of a plant is small, when product demand ($a$ or $L$) and transport cost $\tau$ are large; whereas pure exporter equilibria occur in the opposite configuration. When these conditions are not met, the equilibrium includes at least some exporters and some multinationals.

We conclude this section by a remark on the level of subsidies. In configuration (i) with only exporters, subsidies are always positive and independent of fixed capital costs. However, in configuration (ii) including multinationals, subsidies may be positive or negative depending on the cost of a plant. *Capital may be taxed in the equilibrium of the subsidy game* when fixed costs are small or when product demand is large. Stated differently, multinational firms represent an immobile tax base which governments may profitably tax, provided being multinational is profitable enough. We can further show the following result:

**Proposition 5 (lower subsidies)** Whenever a subsidy equilibrium with multinationals exists, the subsidies are lower than in the hypothetical case without multinationals.

**Proof.** It is readily verified that $s^*-\bar{s}>0$ if and only if

$$f < \overline{f} \equiv \frac{(b + c)\tau L[(4a(6b + 5c) - (12b^2 + 16bc + 3c^2)\tau)]}{16(2b + c)^2}. \tag{26}$$

As shown in Appendix 3, a subsidy equilibrium with multinationals exists if and only if $f < f_2$. One can check that $\overline{f} - f_2 > 0$ for all admissible parameter values of the model. Hence, condition (26) holds, which proves our claim. ■

Proposition 5 states that, when compared to a world in which multinationals are prohibited, the presence of multinationals relaxes the problem of tax competition, thereby *alleviating the ‘race to the bottom’.*

5 Discussion

We now discuss the impact of market integration and trade barriers on subsidy competition, and we analyze the robustness of our findings with respect to our assumption on the distribution of profits.
5.1 Market integration and trade barriers

In many regions of the world, countries have engaged in a process of market integration under which they have reduced trade barriers and trade costs. While market integration allows consumers to purchase more varieties at lower prices, it also reshapes the spatial structure of industries and alters governments’ incentives to attract firms (see Krugman, 1991). In this section, we show that a fall in trade barriers makes a pure exporter configuration more likely and has ambiguous effects on subsidy competition.

First, a fall in trade barriers makes a pure exporter configuration more likely. Indeed, from Figure 1, one can observe that the set of subsidies supporting the pure exporter configuration (i) gets larger for small trade barriers \( \tau \), whereas the set of subsidies supporting the mixed configuration (ii) shrinks. Indeed, smaller trade barriers shift the border (8) outwards and the borders (9) and (10) inwards. In particular, when \( \tau \) is close to 0, the borders (9) and (10) collapse on the \( 45^\circ \) line so that the mixed configuration (ii) vanishes. One can show that the equilibrium involves only exporters and that subsidy are nil \( (s_H^* = s_F^* = 0) \). Indeed, consumers have the same access to all varieties and firms have the same access to all consumers independently of their production place. So, firms have no incentives to set up a second plant and governments have no incentives to offer subsidies to attract production to their countries.

Second, a fall in trade barriers has ambiguous effects on equilibrium subsidies in the pure exporter configuration. It is readily verified that

\[
\frac{\partial s_H^*}{\partial \tau} \geq 0 \iff \tau \leq \overline{\tau} = \frac{2a(b + c)}{2b^2 + 4bc + c^2},
\]

where \( \overline{\tau} < \tau_{\text{trade}} \). Hence, when trade barriers lie in the interval \((0, \overline{\tau})\), equilibrium subsidies fall with any decrease in trade barriers. This is because the gain of having local firms falls with smaller trade costs, which reduces governments’ incentives to offer subsidies to attract them. By contrast, for trade barriers lying in the interval \((\overline{\tau}, \tau_{\text{trade}})\), equilibrium subsidies fall with any rise in trade barriers. In this case, import prices are high and import demands are small. Therefore, equilibrium subsidies will be largest for intermediate values of trade barriers since the gain from having local production is largest there.

Finally, a fall in trade barriers raises equilibrium subsidies in the mixed configuration. It is readily verified that

\[
\frac{\partial \bar{s}_H}{\partial \tau} = -\frac{L(b + c)(2a - 2b\tau + c\tau)}{16(2b + c)} < 0
\]

where the last inequality holds because of the trade feasibility condition (3). Lower trade barriers decrease the returns to being multinational and entice more firms to export their goods from a single plant. As more firms become exporters, the tax base becomes
more mobile and governments have stronger incentives to subsidize capital. Subsidy competition therefore gets stronger as trade barriers fall.

**Proposition 6 (trade barriers)** In a pure exporter equilibrium, a fall in trade barriers reduces equilibrium subsidies when trade barriers are sufficiently low, whereas it increases them when trade barriers are sufficiently large. In a mixed configuration, a fall in trade barriers always increases subsidies.

### 5.2 Profit distributions

Until now, we have assumed that all profits were equally redistributed to the residents of the two countries. In this section, we relax this assumption to check the robustness of our main results. We firstly show that, although the qualitative properties of the best response functions remain unchanged, the structure of some equilibria strongly differ when profits partly flow to absentee shareholders. Furthermore, we show that regional asymmetries in firms’ ownership are reflected in asymmetries in governments’ subsidies and in the distribution of economic activity.

We firstly derive the optimal subsidies when a share $0 \leq \lambda_H < 1$ of profits accrue to country $H$’s residents, whereas a share $0 \leq \lambda_F < 1$ goes to country $F$’s residents. The total share of profits accruing to countries $H$ and $F$ is equal to $\lambda_H + \lambda_F \leq 1$, with a share $1 - \lambda_H - \lambda_F$ of profit going to absentee shareholders. Without loss of generality, we consider that country-$H$ residents have a larger claim on profits: $\lambda_H \geq \lambda_F$. Redefine welfare (15) in country $H$ as follows:

$$W_H = \frac{L}{2} \Pi_H + \lambda_H \left[ n_H (\Pi_H^x + s_H) + n_F (\Pi_F^x + s_F) + m (\Pi^m + s_H + s_F) \right] - (n_H + m) s_H.$$  

Computations analogous to those in the previous section show that the best response in the support of pure exporter configurations (i) is given by

$$\hat{s}_H(s_F) = \frac{8b + c(5 - 4\lambda_H)}{16b + c(9 - 4\lambda_H)} s_F$$

$$+ \frac{L(b + c)\tau (2a(b + c) - (b^2 + c(3 - 2\lambda_H)b + c^2(1 - \lambda_H))) \tau}{(2b + c)(16b + c(9 - 4\lambda_H))},$$

which has still a positive slope less than unity and a positive intercept. A symmetric expression holds for country $F$. The equilibrium of the subsidy competition game can then be readily computed and depends quite naturally on the shares of profit claims $\lambda_H$ and $\lambda_F$. Of course, equilibrium subsidies are no longer symmetric because of the asymmetric profit claims.
In the case of a mixed configuration (ii), the equilibrium subsidy is given by

\[ s_H = \frac{4b + c(3 - 2\lambda_H)}{8b + c(5 - 2\lambda_H)} f - L(b + c)\tau \frac{2a - (b + c(\lambda_H - 1))\tau}{4(8b + c(5 - 2\lambda_H))}, \]

which is again independent of \( s_F \). This shows that the strategic independence of subsidies in the mixed configuration is not an artifact of our equal profit redistribution assumption.

**Absentee shareholders:** We now show that the structure of multiple equilibria varies with the structure of profit claims. To fix this idea assume, for simplicity, that all profits go to absentee shareholders: \( \lambda_H = \lambda_F = 0 \). We can readily derive the new expressions of the same thresholds \( f_1, f_2 \) and \( f_3 \) as in Proposition 4 and compute the equilibria of the subsidy game. However, the new thresholds now satisfy the conditions \( f_1 < f_3 < f_2 \), which yields the following subsidy equilibria:

**Proposition 7 (subsidy equilibria with absentee shareholders)** Suppose that profits fully accrue to absentee shareholders \( \lambda_H = \lambda_F = 0 \). Then the equilibrium of the subsidy game yields

- a pure multinational configuration if and only if \( f \leq f_1 \);
- a mixed configuration with multinationals and exporters if and only if \( f \in (f_1, f_3) \);
- either a mixed configuration or a pure exporter configuration if and only if \( f \in [f_3, f_2) \);
- a pure exporter configuration if and only if \( f \geq f_2 \).

**Proof.** See Appendix 4. ■

Proposition 6 illustrates the interesting situation in which there exist multiple equilibria in the subsidy game, which are associated with different organizational structures of firms. As already pointed out in Section 3, the best response functions are likely to display an upward jump between configurations (i) and (ii) when governments put no weight on profits. As can be seen from panel (b) of Figure 2, governments’ best response functions have an upward jump when the equilibrium switches from configuration (i) to (ii), which yields multiple equilibria for \( f \in [f_3, f_2) \).

The existence of an upward jump in the best response function for \( f \in [f_3, f_2) \) is related to our observation in Section 3.1: the impact of a subsidy increase in terms of local production is stronger in mixed configurations (ii) than in pure exporter configurations (i). Thus, when governments do not value profits, they set higher subsidies in the presence of multinationals simply because subsidies generate more consumer surplus by driving prices down. This results in two possible equilibria: one equilibrium with high subsidies if the
industry includes both exporters and multinationals, and another equilibrium with small subsidies if the industry includes only exporters. Furthermore, this situation reverses the conclusion of Proposition 5, which was established under the assumption that residents have equal and full profit claims. Here, whenever a subsidy equilibrium with multinationals exists, the subsidies are not lower than in the possible case without multinationals. Instead, multinationals lead to higher equilibrium subsidies. Nevertheless, one can check that welfare may still a priori be higher in both countries in the high subsidy equilibrium, since prices are lower (competition) and there are transport cost savings.

Ownership asymmetries: We now study the impact of countries’ shares in total profits on the equilibrium subsidies. One can compute the equilibrium subsidies \( s^*_H \) and verify that \( \partial s^*_H / \partial \lambda_H > 0 \), whereas \( \partial \bar{s}_H / \partial \lambda_H < 0 \), symmetric conditions holding for country \( F \).

Proposition 8 (profit shares)  
(i) An increase in country \( i \)'s profit share \( \lambda_i \) raises equilibrium subsidies in the pure exporter configurations, whereas it reduces them in the mixed configurations. (ii) The subsidy gap \( s^*_H - \bar{s}_F \) in pure exporter configurations falls with smaller trade costs, whereas the subsidy gap \( \bar{s}_H - \bar{s}_F \) in mixed configurations widens with smaller trade costs if \( \tau < 2a/(10b + 3c) < \tau_{\text{trade}} \), and falls otherwise.

Proof. See Appendix 5.

A direct consequence of the first part of Proposition 8 is that the country with the larger profit share subsidizes more (or taxes less) in the pure exporter equilibrium, whereas it subsidizes less (or taxes more) in a mixed equilibrium.

When all firms are exporters, subsidies are used to attract firms and increase firms’ profits after subsidy. Because the country with larger profit claims recoups a larger share of subsidies, that country is naturally enticed to set higher subsidies. Things are different in the mixed configuration. Indeed, as argued in Section 4 and shown in Proposition 2, profits decrease with the level of subsidies in the mixed configurations. Hence, the country having the higher profit claims will reduce its subsidy, since doing so will increase its residents’ wealth.

The second part of Proposition 8 states that regional asymmetries in subsidy (or tax) policies vanish as trade costs fall in pure exporter configurations whereas those asymmetries may amplify in the presence of multinationals. As explained in the previous paragraph, in pure exporter configurations, a subsidy is less costly for the country having higher profit claims because residents recoup a larger part of it through profit claims. In mixed configurations, the country with higher profit claims subsidizes less in order to save on subsidy expenses and to boost profits, which largely go to its domestic shareholders.

Finally, note that in the pure exporter case, international integration reduces the tax gap between countries. This is because, contrary to Baldwin and Krugman (2004), there
are no agglomeration economies that can be taxed away in our model and because firms are not fully agglomerated in one country. Yet, in the presence of multinationals, the subsidy gap widens for sufficiently small values of trade costs $\tau$. In this case, subsidies in both countries fall, yet they fall faster in the country having lower profit claims.

To conclude this section, we note that the shape of governments’ best responses does not depend on how profits are shared within the two countries and between the two countries and the rest of the world. Of course, the equilibria become asymmetric and take different values when profit ownership is not symmetric, yet the general nature of the regimes remains the same. This suggests that the findings of Section 4 are robust.

6 Conclusions

In this paper, we have developed a model of subsidy competition when firms may endogenously choose to become horizontal multinationals. We have shown that the impact of subsidies on prices and profits significantly differ when firms are able to choose their location and organizational structure. In particular, when the cost of a second plant is large relative to the costs of exporting goods, firms prefer to export their goods from a single location only. Hence, when a country raises its subsidy, firms relocate their plant to that country so that competition is strengthened there and is relaxed in the other country. Yet, the impact of the subsidy and the associated regional shift in competition translates into higher subsidy-inclusive profits for the industry. On the contrary, when the cost of a second plant is small relative to the costs of exporting goods, some firms prefer to operate a plant in every country. In that case, when the subsidy in a country rises, some exporters choose to open a second plant in this country so that competition is exacerbated there, without getting relaxed in the other country. Competition increases globally so that prices fall. In contrast to the case with exporting firms, this maps into a fall in subsidy-inclusive profits. Thus, subsidies may trigger a rise or a fall in subsidy-inclusive profits, depending on firms’ changes in operational structure in response to the subsidy.

Governments’ equilibrium subsidies are affected by firms’ locational and organizational choices. In the absence of horizontal multinational firms, firms locate according to subsidy differences and competition for mobile capital entices governments to inflate subsidies. This is the traditional result of the tax competition literature. However, the government may not be responsive to the subsidy of the other governments in the presence of horizontal multinational firms. Indeed, as higher subsidies reduce the cost of capital, firms are enticed to set up additional plants. In this case, an increase or a decrease in a country’s subsidy induces some firms to open or to shut down a plant in that country only, and this decision does not affect the production and price structure in the other country. The subsidy only
has a local impact and induces no response from the other country. Subsidy competition (or tax competition) for mobile capital is therefore less fierce. This finding may shed some light on why the prediction of the ‘race to the bottom’ lacks empirical support, despite the increasing presence of multinationals in the world economy.

We also discussed the case of trade integration and the role of firms’ ownership on subsidies and equilibrium structures. We have shown that a fall in trade barriers has no unambiguous effect on subsidies in equilibria where all firms choose to export from a single location only. By contrast a fall in trade barriers raises subsidies when countries host multinational firms. Finally, when profits leak out of the countries where firms locate, we have shown that there may exist multiple equilibria: the first equilibrium involves only exporting firms at low subsidies, whereas the second involves a combination of exporters and horizontal multinationals at high subsidies. Although the welfare effect is ambiguous, it is actually possible that the high subsidy equilibrium dominates the low subsidy equilibrium, since it leads to lower prices and transport cost savings. Stated differently, the welfare impacts of the ‘race to the bottom’ may be less clear-cut in the endogenous presence of multinationals. This aspect is left for future research.

References


Appendix 1: Firms’ structure and location

In addition to configurations (i) and (ii) discussed in Section 3, the following configurations may arise (but are irrelevant for the subsequent subsidy game):

(iii) **Pure multinational configurations** (i.e., \( m^* = 1 \) and \( n^*_H = n^*_F = 0 \)) are supported for subsidies such that \( \Pi^m + s_H + s_F > \Pi^m_H + s_H \) and \( \Pi^m + s_H + s_F > \Pi^m_F + s_F \), i.e., for \( s_F > B \) and \( s_H > B \). Under these conditions, no firm has an incentive to shut down a plant given the current level of subsidies. The set of subsidies supporting this configuration is depicted by area (iii) in Figure 1.

(iv) **Mixed configurations with agglomeration in** \( H \) (i.e., \( n^*_F = 0, n^*_H > 0 \), and \( m^* > 0 \)) require that subsidy-inclusive profits of exporters in \( H \) and multinationals are equalized: \( \Pi^m_H + s_H = \Pi^m + s_H + s_F \). This yields

\[
n^*_H = \frac{B - s_F}{K} \quad \text{and} \quad m^* = 1 - n^*_H.
\]

Feasibility of this configuration further requires that \( n^*_H > 0, m^* \geq 0 \) and \( \Pi^m_H + s_H > \Pi^m_F + s_F \), which implies that \( B - K \leq s_F \leq B \) and \( s_H > B \). The set of subsidies supporting this configuration is depicted by area (iv) in Figure 1.

(v) **Full agglomeration in** \( H \) (i.e., \( n^*_H = 1, n^*_F = m^* = 0 \)) is feasible when \( \Pi^m_H + s_H > \Pi^m_F + s_F \) and \( \Pi^m_H > \Pi^m_F + s_F \), which implies that \( s_F < s_H - K \) and \( s_F < B + K \). The set of subsidies supporting this configuration is depicted by area (v) in Figure 1.

Note, finally, that there exist by symmetry two additional configurations \((iv')\) and \((v')\) that are the mirror cases of (iv) and (v), namely mixed configurations with exporters in \( F \) only, and full agglomeration in \( F \).

Appendix 2: Best responses

In this appendix, we derive the optimal subsidies for configurations (i) and (ii). First, we compute the following expressions

\[
\frac{\partial S_H}{\partial n_H} = -\frac{1}{2}p_{HH}[2a - (b + c)p_{HH}b + 2p_{HH}c(n_H + n_F) + cn_F\tau]
\]

\[
\frac{\partial S_H}{\partial n_F} = -\frac{1}{8}[(\tau + 2p_{HH})[-2(b + c)p_{HH}b + 4p_{HH}c(n_F + n_H)] - c\tau - b\tau + 4a + 2cn_F\tau],
\]

which we will use in both cases.
Best responses in the support of configuration (i): In the pure exporter case, some straightforward calculations show that

\[
n_H \frac{\partial \Pi_H}{\partial p_{HH}} = n_H (b + c) L p_{HH}, \quad n_F \frac{\partial \Pi_F}{\partial p_{HH}} = \frac{1}{2} n_F L (b + c) (2p_{HH} - \tau)
\]

\[
n_H \frac{\partial \Pi_H}{\partial p_{FF}} = \frac{1}{2} n_H L (b + c) (2p_{FF} - \tau), \quad n_F \frac{\partial \Pi_F}{\partial p_{FF}} = n_F (b + c) L p_{FF}
\]

and

\[
\frac{\partial S_H}{\partial p_{HH}} = -a + b p_{HH} + \frac{1}{2} n_F \tau b.
\]

Using these intermediate results, as well as the equilibrium prices (1) and the equilibrium distribution of firms (7), we can compute the following expressions:

\[
-\frac{1}{2} n_H - \frac{\partial n_H}{\partial s_H} s_H = \left. \frac{\partial S_H}{\partial n_H} \right|_{n_H} - \left. \frac{\partial S_H}{\partial n_F} \right|_{n_F} = \frac{-1}{4K} (K + 3s_H - s_F)
\]

\[
\left. \frac{\partial S_H}{\partial n_H} \right|_{n_H} - \left. \frac{\partial S_H}{\partial n_F} \right|_{n_F} = \frac{\tau (2a - \tau b) b + c}{4} - \frac{s_H - s_F}{2b + c}
\]

\[
L \left. \frac{\partial \Pi_H}{\partial p_{HH}} \right|_{n_H} + n_F \left. \frac{\partial \Pi_F}{\partial p_{HH}} \right|_{n_F} = (2b + c) L \left[ p_{HH} - \frac{2a + cn_F \tau}{2(2b + c)} \right] = 0
\]

\[
\frac{1}{2} \left. \frac{\partial \Pi_H}{\partial p_{FF}} \right|_{n_H} + \frac{1}{2} \left. \frac{\partial \Pi_F}{\partial p_{FF}} \right|_{n_F} = \frac{n_H L (b + c) (2p_{FF} - \tau)}{4} + \frac{n_F (b + c) L p_{FF}}{2}
\]

\[
- \frac{(s_H - s_F) b}{c \tau} + \frac{L (2a - b \tau) b + c}{4} \frac{2}{2b + c}.
\]

Substituting these expressions into the marginal welfare (18), it is readily verified that \(dW_H/ds_H \geq 0\) if and only if

\[
-2 (2b + c) (16b + 7c) s_H + 2 (8b + 3c) (2b + c) s_F
\]

\[
+ \tau L (b + c) \left( 4a (b + c) - \tau \left( 2b^2 + 4bc + c^2 \right) \right) \geq 0,
\]

which shows that local welfare \(W_H\) is concave in \(s_H\) and that the optimal subsidy is given by \(\hat{s}_H(s_F)\).

Best responses in the support of configuration (ii): In the mixed configurations, we readily obtain

\[
L \left. \frac{\partial \Pi_H}{\partial p_{HH}} \right|_{n_H} - L \left. \frac{\partial \Pi_F}{\partial p_{HH}} \right|_{n_F} = n_H (b + c) L p_{HH}, \quad n_H \left. \frac{\partial \Pi_H}{\partial p_{HH}} \right|_{n_H} = n_H (b + c) L p_{HH}
\]

and

\[
n_F \left. \frac{\partial \Pi_F}{\partial p_{HH}} \right|_{n_F} = \frac{L (b + c)}{2} n_F (2p_{HH} - \tau), \quad \frac{m \Pi_m}{\partial p_{HH}} = mL (b + c) p_{HH}.
\]

This allows us to compute

\[
\frac{L}{2} \left. \frac{\partial \Pi_H}{\partial p_{HH}} \right|_{n_H} + \frac{n_H}{2} \left. \frac{\partial \Pi_H}{\partial p_{HH}} \right|_{n_H} + \frac{n_F}{2} \left. \frac{\partial \Pi_F}{\partial p_{HH}} \right|_{n_F} + \frac{m}{2} \left. \frac{\partial \Pi_m}{\partial p_{HH}} \right|_{m} = (2b + c) L \left[ p_{HH} - \frac{2a + cn_F \tau}{2(2b + c)} \right]
\]

\[
= 0.
\]
\[-\frac{1}{2} (n_H + m) + s_H \frac{\partial n_F}{\partial s_H} = \frac{2 (2b + c)}{L (b + c) \tau^2 c} \left( f - \frac{(b + c)[4a - (2b + c)\tau]}{8(2b + c)} L\tau - 3s_H \right) - \frac{1}{2}
\]
\[L \frac{\partial S_H}{\partial m} - L \frac{\partial S_H}{\partial n_F} = \frac{L\tau (a - bp_{HH})}{2} - \frac{L\tau^2 (b + c - 2n_F c)}{8} = -s_H + f\]

Substituting these expressions into the marginal welfare (20), we obtain
\[
\frac{dW_H}{ds_H} = -8s_H \frac{2b + c}{L (b + c) \tau^2 c} - \frac{L (b + c) \tau (4a - 2\tau b + c\tau) - 16f (2b + c)}{4L (b + c) \tau^2 c},
\]
which shows again that local welfare is concave in \(s_H\) and that the optimal subsidy is given by (21).

**Appendix 3: Subsidy equilibria**

To analyze the subsidy equilibria, we need to characterize two points in the subsidy set. Let \((s^1_H, s^1_F)\) and \((s^2_H, s^2_F)\) be the intersection points of the line \(s_H = 2B - K - s_F\) with the best response functions \(s_H = \bar{s}\) and \(s_H = \tilde{s}_H(s_F)\), respectively. One can check that those points are given by
\[
s^1_H = \bar{s} \quad \text{and} \quad s^1_F = \frac{3}{2} f - \frac{L(b + c)\tau [28a - (14b + c)\tau]}{32(2b + c)}
\]
\[s^2_H = \tilde{s}_H(s^2_F) \quad \text{and} \quad s^2_F = \frac{16b + 7c}{12b + 5c} f - \frac{L(b + c)\tau (18a - 9b\tau - c\tau)}{4(12b + 5c)}.
\]

One can further check that
\[
s^1_F > B - K \iff f > f_1 \equiv \frac{L(b + c)\tau (12a - (6b + 5c)\tau)}{16(2b + c)},
\]
\[s^1_F > B - \frac{K}{2} \iff f > f_2 \equiv \frac{L(b + c)\tau (12a - (6b + c) \tau)}{16(2b + c)},
\]
\[s^2_F > B - \frac{K}{2} \iff f > f_3 \equiv \frac{L(b + c)\tau [4a(3b + 2c) - (6b^2 + 6cb + c^2) \tau]}{8(2b + c)^2}.
\]

Under condition (3), we get \(0 < f_1 < f_2 < f_3\).

We now study the best response functions for four ranges of the fixed cost \(f\). The reader can refer to Figure 3 to check the associated values of the subsidy pairs \((s^1_H, s^1_F)\) and \((s^2_H, s^2_F)\) and to visualize the equilibria.\(^{16}\)

Insert Figure 3 about here.

First, when \(f \leq f_1\), we get \(s^1_F \leq B - K/2\), so that the sole equilibrium lies on the border between configuration (ii) and (iii) and implies only multinationals. We also get

\(^{16}\)The parameter values underlying Figure 3 are the same than the ones in Figure 2, except for the fixed cost \(f\).
\( s_F^2 < B - K/2 \) which implies that there exists no pure exporter equilibrium in the support of configuration (i). The unique equilibrium thus involves only multinationals.

Second, when \( f_1 < f < f_2 \), we get \( B - K < s_F^1 < B - K/2 \) so that there exists an equilibrium in the support of configuration (ii). Furthermore, we still have \( s_F^2 < B - K/2 \) which implies that there exists no pure exporter equilibrium in the support of configuration (i). Therefore, the unique equilibrium involves multinationals and exporters.

Third, when \( f_2 < f < f_3 \), we get \( B - K/2 < s_F^1 \) so that there exists no equilibrium in the support of configuration (ii). Furthermore, we still have \( s_F^2 < B - K/2 \) which implies that there exists no pure exporter equilibrium in the support of configuration (i). Equilibria of the subsidy game belong to the border between configurations (i) and (ii).

There thus exists a continuum of equilibria such that \((s_H, s_F) = (s, 2B - K - s)\) where \( s \in [\max \{s_F^2, 2B - K - s_F^1\}, \min \{s_F^1, 2B - K - s_F^2\}]\).

Last, when \( f \geq f_3 \), we get \( B - K/2 < s_F^1 \) so that there exists no equilibrium in the support of configuration (ii), whereas we now have \( s_F^2 \geq B - K/2 \) which implies the existence of a pure exporter equilibrium in the support of configuration (i). This is the unique equilibrium.

### Appendix 4: Absentee shareholders

As in Appendix 3, we compute the intersection points \((s_H^1, s_F^1)\) and \((s_H^2, s_F^2)\) in the case where \( \lambda_H = \lambda_F = 0 \). This yields

\[
 s_H^1 = \bar{s} \quad \text{and} \quad s_F^1 = \frac{7c + 12b}{8b + 5c} f - \frac{1}{4} L \tau (b + c) \frac{2 (9c + 14b) a - \tau (c^2 + 11cb + 14b^2)}{(8b + 5c) (2b + c)}
\]

\[
 s_H^2 = s_H^1(s_F^2) \quad \text{and} \quad s_F^2 = \frac{9c + 16b}{7c + 12b} f - \frac{1}{4} L \tau (b + c) \frac{2 (11c + 18b) a - \tau (c + 6b) (2c + 3b)}{(2b + c) (7c + 12b)}
\]

Therefore,

\[
 s_F^1 > B - K \iff f > f_1 \equiv \frac{1}{16} (b + c) L \tau \frac{8 (2c + 3b) a - \tau (2b + c) (7c + 6b)}{(2b + c)^2}
\]

\[
 s_F^2 > B - K/2 \iff f > f_2 \equiv \frac{1}{8} (b + c) L \tau \frac{4 (2c + 3b) a - \tau (6cb + c^2 + 6b^2)}{(2b + c)^2},
\]

\[
 s_F^2 > B - K/2 \iff f > f_3 \equiv \frac{1}{4} (b + c) L \tau \frac{2 (2c + 3b) a - \tau (c + 3b) (b + c)}{(2b + c)^2}.
\]

It is readily verified that \( f_1 < f_3 < f_2 \), i.e., when compared to the case with full profit redistribution, the thresholds \( f_2 \) and \( f_3 \) get reversed. The equilibrium conditions are the same as in Appendix 3. Note, on the one hand, that the continuum of equilibria disappears because \( f_3 < f_2 \). On the other hand, the supports of pure exporter equilibria \((f \geq f_3)\) and of mixed equilibria \((f \in (f_1, f_2))\) now overlap. Hence, there exist multiple equilibria for \( f \in (f_2, f_3)\).
Appendix 5: Ownership asymmetries

We firstly prove part (i) of Proposition 8. Some straightforward computations show that

\[
\frac{\partial s_H^*}{\partial \lambda_H} = \frac{cL(b + c)(12b + c(7 - 4\lambda_F))(16b + c(9 - 4\lambda_F))\tau^2}{8(2b + c)(12b + c(-2\lambda_F - 2\lambda_H + 7))^2} > 0.
\]

Some further computations show that

\[
\frac{\partial \tilde{s}_H}{\partial \lambda_H} < 0 \iff f > \frac{(b + c)L\tau((-4a + 10b\tau + 3c\tau)}{16(2b + c)} \equiv \bar{f}. \tag{27}
\]

For a mixed configuration to be feasible in the first place, \( f \) must lie above the threshold \( f_1 \), which in the case of absentee shareholders is now given by:

\[
f_1 = \frac{(b + c)L\tau(8a(3b + c(2 - \lambda_H)) - (2b + c)(6b + c(7 - 4\lambda_H))\tau)}{16(2b + c)^2} \tag{28}
\]

One can check that \( \bar{f} < f_1 \) for all \( 0 \leq \lambda_H \leq 1 \), which allows us to conclude that \( \frac{\partial \tilde{s}_H}{\partial \lambda_H} < 0 \).

We next prove part (ii) of Proposition 8. Some longer computations reveal that, the difference in equilibrium subsidies in the pure exporter case is equal to

\[
s_H^* - s_F^* = \frac{c(b + c)L(\lambda_H - \lambda_F)\tau^2}{24b + 2c(7 - 2\lambda_F - 2\lambda_H)} \geq 0
\]

since we assume that \( \lambda_H \geq \lambda_F \). This expression decreases with smaller trade costs.

Furthermore, in the mixed equilibrium we have

\[
\tilde{s}_H - \tilde{s}_F = \frac{c(\lambda_H - \lambda_F)[(b + c)(10b + 3c)L\tau^2 - 4a(b + c)L\tau - 16(2b + c)f]}{4(8b + c(5 - 2\lambda_F))(8b + c(5 - 2\lambda_H))}
\]

which is negative if and only if \( \bar{f} < f \), where \( \bar{f} \) is defined above. Since \( \bar{f} < f_1 \) and \( f_1 < f \) in mixed configurations, we get \( \tilde{s}_H - \tilde{s}_F < 0 \). It can furthermore be shown that \( \tilde{s}_H - \tilde{s}_F \) increases with smaller trade costs if \( \tau < 2a/(10b + 3c) < \tau_{\text{trade}} \) and falls otherwise.
Figure 1: Domains of the different configurations

Figure 2: Continuous and discontinuous best response functions
Figure 3: Evolution of equilibria for $f = 1$, $f = 1.2$, $f = 1.275$ and $f = 1.4$
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